Perturbations to $\mu - \tau$ symmetry in neutrino mixing

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Abstract

Many neutrino mixing scenarios that have $\mu - \tau$ symmetry with $\theta_{13} = 0$ are in disagreement with recent experimental results that indicate a nonzero value for θ_{13}. We investigate the effect of small perturbations on Majorana mass matrices with $\mu - \tau$ symmetry and derive analytic formulae for the corrections to the mixing angles. We find that since m_1 and m_2 are nearly degenerate, $\mu - \tau$ symmetry mixing scenarios are able to explain the experimental data with about the same size perturbation for most values of θ_{12}. This suggests that the underlying unperturbed mixing need not have θ_{12} close to the experimentally preferred value. One consequence of this is that a new class of models with $\mu - \tau$ symmetry is possible, with unperturbed θ_{12} equal to zero or $90°$ for arbitrary unperturbed θ_{13}.

Of the numerous neutrino mixing scenarios discussed in the literature [1], several have $\mu - \tau$ symmetry, such as tri-bimaximal mixing (TBM) [2], bimaximal mixing (BM) [3], hexagonal mixing (HM) [4] and scenarios of A_5 mixing [5]. In these scenarios, $\theta_{23} = 45°$, $\theta_{13} = 0$, and only θ_{12} depends on the particular model. Tri-bimaximal mixing is most popular because the value of θ_{12} predicted by TBM is close to that preferred by the current experimental data. However, the latest results from the T2K [6], MINOS [7], and Double Chooz [8] experiments suggest a nonzero value of θ_{13}, and the recent Daya Bay [9] and RENO [10] experiments find $\theta_{13} \neq 0$ at the 5.2σ and 4.9σ level, respectively. Various corrections may reconcile such models with nonzero θ_{13} [1]. In this Letter we consider small perturbations acting on Majorana mass matrices with $\mu - \tau$ symmetry and estimate the size of perturbations required to explain the experimental data.

We find that for $\mu - \tau$ symmetries with almost any initial value of θ_{12} (i.e., before the perturbation), the minimal size of the perturbations needed to bring the model in agreement with experimental data varies by only about 20%. The reason is that the θ_{12} correction depends only on the ratio of perturbation terms and not on their absolute size, and the overall size of the perturbation is determined by the corrections to θ_{13} and θ_{23}, which are relatively small. We also show that a new category of models with $\mu - \tau$ symmetry, $\theta_{23} = 45°$, $\theta_{12} = 0$ or $90°$, and arbitrary θ_{13}, can also fit the data with small perturbations.

We start with the mass matrix for Majorana neutrinos

$$M = U^* M^{\rm diag} U^\dagger \,, \tag{1}$$

where $M^{\rm diag} = {\rm diag}(m_1, m_2, m_3)$, U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [11] (without the multiplicative diagonal matrix of Majorana phases), and we work in the basis in which the charged lepton mass matrix is diagonal. The masses m_2 and m_3 are complex and m_1 can be taken to be real and non-negative.

The general condition describing $\mu - \tau$ symmetry (also sometimes called $\mu - \tau$ universality) is [12]

$$|U_{\mu i}| = |U_{\tau i}| \,, \text{ for } i = 1, 2, 3. \tag{2}$$

From the standard form of the mixing matrix these conditions are equivalent to

$$\theta_{23} = 45° \,, \quad {\rm Re}(\cos\theta_{12} \sin\theta_{12} \sin\theta_{13} e^{i\delta}) = 0 \,. \tag{3}$$

Hence, there are four classes of $\mu - \tau$ symmetry: (a) $\theta_{23} = 45°, \theta_{13} = 0$; (b) $\theta_{23} = 45°, \theta_{12} = 0$; (c) $\theta_{23} = 45°, \theta_{12} = 90°$; (d) $\theta_{23} = 45°, \delta = \pm 90°$. Class (a) contains models with tri-bimaximal, bimaximal, hexagonal, and A_5 symmetries, while class (d) includes tetramaximal symmetry [13]. Classes (b) and (c) have not been studied before because the unperturbed θ_{12} angle is far from the experimentally preferred value, but, as we show below, small perturbations can have a large effect on θ_{12}, and therefore these models should not be ignored.

Class (a): $\theta_{23}^0 = 45^\circ$, $\theta_{13}^0 = 0$

We first examine the effect of small perturbations on models in class (a). The initial (unperturbed) mixing matrix can be written as

$$U_0 = \begin{pmatrix} \cos\theta_{12}^0 & \sin\theta_{12}^0 & 0 \\ -\frac{\sin\theta_{12}^0}{\sqrt{2}} & \frac{\cos\theta_{12}^0}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}^0}{\sqrt{2}} & -\frac{\cos\theta_{12}^0}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \qquad (4)$$

and the initial mass matrix is

$$M_0 = U_0^* M_0^{\text{diag}} U_0^\dagger = \begin{pmatrix} m_1^0 c_{12}^2 + m_2^0 s_{12}^2 & \frac{(m_2^0 - m_1^0) s_{12} c_{12}}{\sqrt{2}} & \frac{(m_1^0 - m_2^0) s_{12} c_{12}}{\sqrt{2}} \\ \frac{(m_2^0 - m_1^0) s_{12} c_{12}}{\sqrt{2}} & \frac{1}{2}(m_3^0 + m_2^0 c_{12}^2 + m_1^0 s_{12}^2) & \frac{1}{2}(m_3^0 - m_2^0 c_{12}^2 - m_1^0 s_{12}^2) \\ \frac{(m_1^0 - m_2^0) s_{12} c_{12}}{\sqrt{2}} & \frac{1}{2}(m_3^0 - m_2^0 c_{12}^2 - m_1^0 s_{12}^2) & \frac{1}{2}(m_3^0 + m_2^0 c_{12}^2 + m_1^0 s_{12}^2) \end{pmatrix} , \qquad (5)$$

where $M_0^{\text{diag}} = \text{diag}(m_1^0, m_2^0, m_3^0)$, and c_{jk}, s_{jk} denotes $\cos\theta_{jk}^0$ and $\sin\theta_{jk}^0$ respectively. Under a small perturbation the final (resultant) mass matrix can be written as

$$M = U_0^* M_0^{\text{diag}} U_0^\dagger + E \, , \qquad (6)$$

where the perturbation matrix E has the general form

$$E = M - M_0 = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{pmatrix} . \qquad (7)$$

Treating the three masses as eigenvalues of the mass matrix with each column of the mixing matrix as the corresponding eigenvector, we can use traditional perturbation methods to find the corrections to the three angles and three masses. From experiment we know that m_1 and m_2 are nearly degenerate, so that degenerate perturbation theory with $|\delta m_{21}^0| \ll |\delta m_{31}^0|$ and $|\epsilon_{ij}| < |m_k^0|$ (where $\delta m_{ji}^0 = m_j^0 - m_i^0$, and the index k denotes the heaviest eigenstate), can be used. For simplicity, we assume M_0 and E are real and employ the following notation:

$$\begin{aligned} &\epsilon_1 = \epsilon_{11} \, , \quad \epsilon_2 = \epsilon_{12} + \epsilon_{13} \, , \quad \epsilon_3 = \epsilon_{12} - \epsilon_{13} \, , \quad \epsilon_4 = \epsilon_{22} + \epsilon_{33} + 2\epsilon_{23} \, , \\ &\epsilon_5 = \epsilon_{22} - \epsilon_{33} \, , \quad \epsilon_6 = \epsilon_{22} + \epsilon_{33} - 2\epsilon_{23} - 2\epsilon_{11} \, . \end{aligned} \qquad (8)$$

We find the first order corrections to the three masses to be

$$\delta m_i^{(1)} = \frac{1}{4}\left[4\epsilon_1 + \epsilon_6 \pm \left(2\delta m_{21}^0 - \sqrt{8\epsilon_3^2 + \epsilon_6^2 + 4(\delta m_{21}^0)^2 + 4\delta m_{21}^0 (2\sqrt{2}\epsilon_3 \sin 2\theta_{12}^0 + \epsilon_6 \cos 2\theta_{12}^0)}\right)\right] , \qquad (9)$$

where the plus sign is for $i = 1$ and the minus sign is for $i = 2$, and

$$\delta m_3^{(1)} = \frac{1}{2}\epsilon_4 \, . \qquad (10)$$

Table 1: Best-fit values and 2σ ranges of the oscillation parameters [14] used to find the ϵ_{ij}, with $\delta m^2 \equiv |m_2|^2 - m_1^2$ and $\Delta m^2 \equiv |m_3|^2 - (m_1^2 + |m_2|^2)/2$.

Hierarchy	$\theta_{12}(^\circ)$	$\theta_{13}(^\circ)$	$\theta_{23}(^\circ)$	$\delta m^2(10^{-5}\text{eV}^2)$	$\|\Delta m^2\|(10^{-3}\text{eV}^2)$
Normal	$33.6^{+2.1}_{-2.0}$	$8.9^{+0.9}_{-0.9}$	$38.4^{+3.6}_{-2.3}$	$7.54^{+0.46}_{-0.39}$	$2.43^{+0.12}_{-0.16}$
Inverted	$33.6^{+2.1}_{-2.0}$	$9.0^{+0.8}_{-1.0}$	$38.8^{+5.3}_{-2.3} \oplus 47.5 - 53.2$	$7.54^{+0.46}_{-0.39}$	$2.42^{+0.11}_{-0.16}$

The first order corrections to the mixing angles are

$$\delta\theta_{12}^{(1)} = \frac{1}{2}\arctan\frac{2\sqrt{2}\epsilon_3\cos 2\theta_{12}^0 - \epsilon_6\sin 2\theta_{12}^0}{2\sqrt{2}\epsilon_3\sin 2\theta_{12}^0 + \epsilon_6\cos 2\theta_{12}^0 + 2\delta m_{21}^0}\,, \tag{11}$$

$$\delta\theta_{23}^{(1)} = \frac{\epsilon_5 s_{12}^2 - \sqrt{2}\epsilon_2 s_{12}c_{12}}{2\delta m_{31}^0} + \frac{\epsilon_5 c_{12}^2 + \sqrt{2}\epsilon_2 s_{12}c_{12}}{2\delta m_{32}^0}\,, \tag{12}$$

$$\delta\theta_{13}^{(1)} = \frac{\sqrt{2}\epsilon_2 c_{12}^2 - \epsilon_5 s_{12}c_{12}}{2\delta m_{31}^0} + \frac{\sqrt{2}\epsilon_2 s_{12}^2 + \epsilon_5 s_{12}c_{12}}{2\delta m_{32}^0}\,, \tag{13}$$

and the second order correction to θ_{12} is

$$\delta\theta_{12}^{(2)} = -\frac{\sqrt{2}\epsilon_2\epsilon_5\cos 2(\theta_{12}^0 + \delta\theta_{12}^{(1)}) + (\epsilon_2^2 - \epsilon_5^2/2)\sin 2(\theta_{12}^0 + \delta\theta_{12}^{(1)})}{4\delta m_{21}^0\delta m_{32}^0}\,. \tag{14}$$

Imposing $|\delta m_{21}^0| \ll |\delta m_{31}^0|$, the expressions for $\delta\theta_{23}^{(1)}$ and $\delta\theta_{13}^{(1)}$ simplify to

$$\delta\theta_{23}^{(1)} \simeq \frac{\epsilon_5}{2\delta m_{31}^0}\,, \quad \delta\theta_{13}^{(1)} \simeq \frac{\sqrt{2}\epsilon_2}{2\delta m_{31}^0}\,. \tag{15}$$

We note that while $\delta\theta_{23}^{(1)}$ and $\delta\theta_{13}^{(1)}$ are suppressed by a factor of order $\epsilon_j/\delta m_{31}^0$, to leading order $\delta\theta_{12}$ depends only on ratios of linear combinations of ϵ_3, ϵ_6 and δm_{21}^0 (which is $\mathcal{O}(\epsilon_{ij})$). Therefore large corrections to θ_{12} are possible even for small corrections to θ_{23} and θ_{13}.

A recent global three-neutrino fit [14] yields the parameter values in Table 1. We have done a numerical search to find perturbed mass matrices that give the oscillation parameters *and* which have small perturbations. In our search, we first fix $\theta_{23}^0 = 45^\circ$ and $\theta_{13}^0 = 0$, consistent with $\mu - \tau$ symmetry, and choose a particular value for θ_{12}^0 and the magnitude of m_1 for the normal hierarchy (or m_3 for the inverted hierarchy). The global fit in Table 1 then defines the magnitudes of the other two final masses and the three final mixing angles (since $\theta_{13}^0 = 0$, the initial Dirac phase does not matter).

We characterize the size of the perturbation as the root-mean-square (RMS) value of the perturbations, i.e.,

$$\epsilon_{RMS} = \sqrt{\frac{\sum_{i,j=1}^3 |M_{ij} - M_{0ij}|^2}{9}}\,, \tag{16}$$

Table 2: Top half: values of the perturbations (in 10^{-3} eV) that give the best-fit parameters in Table 1 *and* have the minimum ϵ_{RMS} for the given θ_{12}^0, for the normal hierarchy and $m_1 = 0$. Bottom half: representative values that fit the experimental data within 2σ and for which all ϵ_{ij} have a similar magnitude (with $m_1^0 = 0$, $m_2^0 = 0.0054$ eV, $m_3^0 = 0.0595$ eV, $m_1 = 0.0072$ eV, $\delta = 180°$ and all other phases equal to 0).

$\theta_{12}^0(°)$	ϵ_{11}	ϵ_{12}	ϵ_{13}	ϵ_{22}	ϵ_{23}	ϵ_{33}	ϵ_{RMS}
60	-3.05	-3.50	-5.99	-2.72	-1.52	5.77	4.10
45 (BM)	-1.32	-4.74	-4.74	-3.58	-0.66	4.90	3.79
35.3 (TBM)	0.32	-4.66	-4.82	-4.40	0.16	4.08	3.74
30 (HM)	1.07	-4.31	-5.18	-4.78	0.54	3.71	3.79
0	0.00	-1.38	-8.11	-4.24	0.00	4.24	4.36
60	5.41	-4.17	-4.52	-5.00	-9.94	3.36	6.14
45 (BM)	6.76	-4.43	-4.26	-5.67	-9.27	2.69	6.08
35.3 (TBM)	7.66	-4.32	-4.37	-6.12	-8.82	2.24	6.08
30 (HM)	8.11	-4.17	-4.52	-6.35	-8.59	2.01	6.09
0	9.46	-2.52	-6.17	-7.02	-7.92	1.34	6.28

where i and j sum over neutrino flavors. Hence, ϵ_{RMS} is determined by the following quantities: three initial masses, two initial Majorana phases, two final Majorana phases and one final Dirac phase. We scan over these quantities with all phases taken to be either 0 or 180° to find the minimum value of ϵ_{RMS} for a given θ_{12}^0. We follow the same procedure for classes (b) and (c) below. For class (d), all values of the phases are allowed.

We show the perturbations that give the smallest ϵ_{RMS} for the normal hierarchy, $m_1 = 0$ and several values of θ_{12}^0 in Table 2. It is clear that the sizes of ϵ_{RMS} are approximately the same regardless of the value of θ_{12}^0; we find that the smallest ϵ_{RMS} for each θ_{12}^0 varies by at most 17% for the examples shown. This can be explained by the perturbation results derived above as follows. From Eq. (8) we have $\epsilon_{RMS} = \sqrt{\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \frac{1}{2}\epsilon_5^2 + \frac{1}{4}\epsilon_4^2 + \frac{1}{4}(2\epsilon_1 + \epsilon_6)^2}/3$; since $m_3 \gg m_1, m_2$ for the normal hierarchy with $m_1 = 0$ eV and the first order perturbations of the three masses are much smaller than m_3, we can assume $\delta m_{31}^0 \approx m_3^0 \approx m_3 \approx \sqrt{\Delta m^2} = 0.0493$ eV. Then from Eq. (15) we know that in order to get the correction $\delta\theta_{23} = -6.6°$ and $\delta\theta_{13} = 8.9°$ for any value of θ_{12}^0, we need $\epsilon_5 = -0.0114$ eV and $\epsilon_2 = 0.0108$ eV, so that $\sqrt{\epsilon_2^2 + \epsilon_5^2/2}/3 = 0.00449$ eV, which is already close to the ϵ_{RMS} values found in Table 2. The small discrepancy can be explained by the perturbation of the three masses and other ϵ's. Hence, we can say that the size of the perturbation mainly comes from the corrections to θ_{23} and θ_{13}. From Eq. (11) we know that the correction to θ_{12} is determined by the relative ratio of ϵ_3 to ϵ_6 and the actual size of the perturbation does not matter. This means that we can have large corrections for θ_{12}^0 with a (relatively) small perturbation.

We note that initial values of θ_{12} on the "dark side" ($\theta_{12}^0 > 45°$ and $m_1^0 < m_2^0$) can also fit the data with perturbations that are similar in magnitude to those needed for tri-bimaximal mixing (see the entry for $\theta_{12}^0 = 60°$ in Table 2).

Table 3: Top half: same as Table 2, except for the inverted hierarchy and $m_3 = 0$. Bottom half: same as Table 2, except for the inverted hierarchy and $m_1^0 = 0.05$ eV, $m_2^0 = 0.052$ eV, $m_3^0 = 0$, $m_3 = 0.002$ eV, and all phases equal to 0.

$\theta_{12}^0(^\circ)$	ϵ_{11}	ϵ_{12}	ϵ_{13}	ϵ_{22}	ϵ_{23}	ϵ_{33}	ϵ_{RMS}
60	-0.86	-4.94	-5.64	5.57	-0.43	-4.72	4.31
45 (BM)	-0.47	-5.29	-5.29	5.38	-0.23	-4.91	4.29
35.3 (TBM)	-0.05	-5.30	-5.28	5.17	0.03	-5.12	4.28
30 (HM)	0.16	-5.23	-5.36	5.07	0.08	-5.22	4.28
0	0.00	-4.47	-6.12	5.15	0.00	-5.15	4.32
60	-3.56	-4.89	-5.27	5.95	2.67	-3.92	4.49
45 (BM)	-3.06	-4.98	-5.18	5.70	2.92	-4.17	4.47
35.3 (TBM)	-2.73	-4.94	-5.22	5.54	3.08	-4.34	4.46
30 (HM)	-2.56	-4.89	-5.27	5.45	3.17	-4.42	4.46
0	-2.06	-4.28	-5.88	5.20	3.42	-4.67	4.50

In the top half of Table 2, ϵ_{11} and ϵ_{23} are much smaller than the other ϵ_{ij} for some values of θ_{12}^0. We have checked that if these values are set to zero, the experimental constraints can still be satisfied at the 2σ level without a large change in the nonzero parameters. Therefore if some perturbations are exactly zero due to symmetries, the resulting mass matrix can still fit the experimental data with small perturbations.

For the inverted hierarchy, some representative sets of ϵ_{ij} that give the minimum ϵ_{RMS} are shown in Table 3 for $m_3 = 0$. The minimum ϵ_{RMS} as a function of θ_{12}^0 varies only by about 1% in this case, i.e., the minimum ϵ_{RMS} varies with θ_{12}^0 even less for the inverted hierarchy than for the normal hierarchy.

Clearly, if perturbations are large enough that tri-bimaximal mixing can explain the experimental data, then other $\mu - \tau$ mixing scenarios, such as bimaximal, hexagonal mixing and A_5 mixing, can also explain the experimental data with about the same size perturbation. Hence, tri-bimaximal mixing has no special position among the $\mu - \tau$ symmetry mixing scenarios when a perturbation is required to fit the experimental data. Also, it is possible for all the perturbations to have a similar magnitude and still give the oscillation parameters within their 2σ ranges; see the bottom half of Tables 2 and 3.

We also varied the size of the final masses by changing the value of m_1 in the normal hierarchy and m_3 in the inverted hierarchy. We find that the minimum ϵ_{RMS} decreases as the size of the final masses increases for both the normal and inverted hierarchies. For the quasi-degenerate hierarchy (in which the magnitude of the absolute masses is larger than $\sqrt{\Delta m^2}$) the size of the perturbation can be very small. This can be explained by the perturbation equations: since $\delta m_{31}^0 \approx m_3 - m_1 \approx \Delta m^2/(m_3 + m_1)$ for small perturbations, and Δm^2 is fixed by experimental data, then δm_{31}^0 will decrease if the masses increase, and similarly for δm_{32}^0. Then Eqs. (12) and (13) show that in order to get the same corrections for θ_{13}^0 and θ_{23}^0, the size of the perturbation should also decrease.

Classes (b) and (c): $\theta_{23}^0 = 45°, \theta_{12}^0 = 0$ or $90°$

For class (b) ($\theta_{23}^0 = 45°$, $\theta_{12}^0 = 0$), since the Dirac phase is irrelevant, the initial mixing matrix and mass matrix can be written as

$$U_0 = \begin{pmatrix} \cos\theta_{13}^0 & 0 & \sin\theta_{13}^0 \\ -\frac{\sin\theta_{13}^0}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\cos\theta_{13}^0}{\sqrt{2}} \\ -\frac{\sin\theta_{13}^0}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{\cos\theta_{13}^0}{\sqrt{2}} \end{pmatrix}, \tag{17}$$

and the initial mass matrix is

$$M_0 = U_0^* M_0^{\mathrm{diag}} U_0^\dagger =$$
$$\begin{pmatrix} m_1^0 c_{13}^2 + m_3^0 s_{13}^2 & \frac{(m_3^0 - m_1^0)s_{13}c_{13}}{\sqrt{2}} & \frac{(m_3^0 - m_1^0)s_{13}c_{13}}{\sqrt{2}} \\ \frac{(m_3^0 - m_1^0)s_{13}c_{13}}{\sqrt{2}} & \frac{1}{2}(m_2^0 + m_3^0 c_{13}^2 + m_1^0 s_{13}^2) & \frac{1}{2}(-m_2^0 + m_3^0 c_{13}^2 + m_1^0 s_{13}^2) \\ \frac{(m_3^0 - m_1^0)s_{13}c_{13}}{\sqrt{2}} & \frac{1}{2}(-m_2^0 + m_3^0 c_{13}^2 + m_1^0 s_{13}^2) & \frac{1}{2}(m_2^0 + m_3^0 c_{13}^2 + m_1^0 s_{13}^2) \end{pmatrix}. \tag{18}$$

If we redefine the phase of the wavefunction ψ_3 to $-\psi_3$, or change the initial angle θ_{23}^0 from 45° to 135° and switch the indices 2 and 3, then the mass matrix in Eq. (18) is exactly the same as that in Eq. (5).

For the above initial mass matrix, corrections must shift θ_{12} from 0 to 33.6°, and θ_{13} from the initial arbitrary angle to 9.0°. We used the same scan procedure as before and searched for the minimum ϵ_{RMS} for various values of θ_{13}^0 (see Table 4). We find that for $\theta_{13}^0 < 20°$, the data can be explained with about the same size perturbation as was found for class (a). For example, when $\theta_{13}^0 = 0$ for class (b), the initial mass matrix is the same as $\theta_{12}^0 = 0$ for class (a), and therefore the minimum ϵ_{RMS} is also the same. In particular, when θ_{13}^0 is close to 9.0° in class (b), the minimum ϵ_{RMS} is even smaller than the minimum value for class (a) because the correction to θ_{13} is smaller in this case. Although the correction to θ_{12} is large, it does not affect the size of the perturbation too much because its size is mainly determined by the corrections to θ_{13} and θ_{23}, as noted before. However, for $\delta\theta_{13}$ greater than about 20°, the size of the perturbation required to fit the data becomes larger since θ_{13} must change by more than 10°.

For class (c) ($\theta_{23}^0 = 45°$, $\theta_{12}^0 = 90°$), we find that switching m_1^0 with m_2^0 makes the initial mass matrix the same as the initial mass matrix of class (b). Since we scan all possible values of m_1^0 and m_2^0, the minimum ϵ_{RMS} for a given θ_{13}^0 for class (c) is the same as for class (b).

Class (d): $\theta_{23}^0 = 45°, \delta^0 = \pm 90°$

If we fix $\theta_{23}^0 = 45°$, $\delta^0 = \pm 90°$ and vary both θ_{12}^0 and θ_{13}^0, this category includes mixing scenarios such as the tetramaximal mixing pattern ($\mathrm{T}^4\mathrm{M}$) [13], and the correlative mixing pattern with $\delta = \pm 90°$ [15]. For $\theta_{13}^0 < 20°$ and $\theta_{12}^0 \leq 45°$, the smallest ϵ_{RMS} for the normal hierarchy (with $m_1 = 0$) varies from 2.29×10^{-3} eV to 5.26×10^{-3} eV, where the minimum value occurs at $\theta_{13}^0 = 9°$ and $\theta_{12}^0 = 32°$, and the maximum value occurs at $\theta_{13}^0 = 20°$ and

Table 4: Top half: same as Table 2, except for class (b) ($\theta_{12}^0 = 0$). Bottom half: same as Table 2, except for class (b).

$\theta_{13}^0(^\circ)$	ϵ_{11}	ϵ_{12}	ϵ_{13}	ϵ_{22}	ϵ_{23}	ϵ_{33}	ϵ_{RMS}
0	0.00	-1.38	-8.11	-4.24	0.00	4.24	4.36
5	0.48	1.44	-5.28	-4.48	-0.24	4.00	3.27
10	-0.44	4.21	-2.52	-4.02	0.22	4.46	3.06
15	-2.64	6.59	-0.14	-2.92	1.32	5.56	3.90
20	-5.85	8.30	1.57	-1.32	2.93	7.17	5.24
0	9.46	-2.52	-6.17	-7.02	-7.92	1.34	6.28
5	9.01	1.13	-2.52	-6.80	-7.69	1.56	5.41
10	7.66	4.67	1.02	-6.12	-7.02	2.24	5.22
15	5.47	8.00	4.35	-5.03	-5.93	3.33	5.80
20	2.50	11.00	7.35	-3.54	-4.44	4.82	6.92

$\theta_{12}^0 = 0$. Therefore small perturbations can fit the experimental data for a wide range of θ_{12}^0 and θ_{13}^0 for class (d).

In summary, we studied small perturbations to Majorana mass matrices with $\mu - \tau$ symmetry that yield experimentally preferred oscillation parameters. We find that the size of the perturbations (which decreases as the neutrino mass scale is increased), is mainly determined by the corrections to θ_{23} and θ_{13}, and that small perturbations can give a very large correction to θ_{12} because to first order, the θ_{12} correction depends only on the ratio of perturbation terms and not on their absolute size. Hence, most mixing scenarios with $\mu - \tau$ symmetry can explain the experimental data with perturbations of similar magnitude, and tri-bimaximal mixing has no special place among scenarios with $\mu - \tau$ symmetry. We also find that slightly perturbed $\mu - \tau$ symmetric models with $\theta_{12} = 0$ or 90° are viable for $\theta_{13} < 20^\circ$.

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